

The $B \rightarrow D_s^{(*)}\pi$ decays in the perturbative QCD

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Abstract

In this paper, we calculate the branching ratios for $B^0 \rightarrow D_s^+\pi^-$, $B^+ \rightarrow D_s^+\pi^0$, $B^0 \rightarrow D_s^{*+}\pi^-$ and $B^+ \rightarrow D_s^{*+}\pi^0$ decays in the perturbative QCD factorization approach. We find that the calculated branching ratios of these four decay channels agree well with the measured values and current experimental upper limit. In the numerical calculation, we take the decay constant and the shape parameter of the vector meson D_s^* as $f_{D_s^*} = 312$ MeV and $a_{D_s^*} = 0.78$ respectively, which are larger than those in the previous calculations.

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I. INTRODUCTION

In recent years, more and more effort has been made to the B meson decays with one [1] even two [2] charmed mesons in the final states and it is found that the perturbative QCD factorization (pQCD) approach does work well in these decays. We will calculate the branching ratios for the $B \rightarrow D_s^{(*)}\pi$ decays, which are shown in figure 1, by employing the pQCD approach. The momenta of the two outgoing mesons are both approximately $\frac{1}{2}m_B(1 - m_{D_s^{(*)}}^2/m_B^2)$. This is still large enough to make a hard intermediate gluon in the hard part calculation. Most of the momenta come from the heavy b quark in quark level. The light quark u (d) inside B^+ (B^0) meson, which is usually called spectator quark, carries small momentum of order of Λ_{QCD} . In order to form a fast moving light meson, the spectator quark need to connect the four-quark operator $(\bar{b}u)_{V-A}(\bar{c}s)_{V-A}$ through an energetic gluon. The hard four-quark dynamic together with the spectator quark becomes six-quark effective interaction. Since six-quark interaction is hard dynamics, it is perturbatively calculable in theory.

On the experimental side, the branching ratios of $B^0 \rightarrow D_s^+\pi^-$, $B^+ \rightarrow D_s^+\pi^0$ and $B^0 \rightarrow D_s^{*+}\pi^-$ have been measured by BaBar [3] and Belle [4]. For $B^+ \rightarrow D_s^{*+}\pi^0$ decay, only the experimental limit is given by CLEO [5]. We list their values in the following [6]:

$$\begin{aligned} Br(B^0 \rightarrow D_s^+\pi^-) &= (1.53 \pm 0.35) \times 10^{-5}, \\ Br(B^+ \rightarrow D_s^+\pi^0) &= (1.6 \pm 0.6) \times 10^{-5}, \\ Br(B^0 \rightarrow D_s^{*+}\pi^-) &= (3.0 \pm 0.7) \times 10^{-5}, \\ Br(B^+ \rightarrow D_s^{*+}\pi^0) &< 2.7 \times 10^{-4}. \end{aligned} \tag{1}$$

This paper is organized as follows. In Sect.II, the light-cone wave functions of the initial and the final state mesons are discussed. In Sec.III, we calculate analytically the related Feynman diagrams and present the various decay amplitudes for the studied decay modes. The numerical results and the discussions are given in the section IV. The conclusions are presented in the final part.

II. WAVE FUNCTIONS OF INITIAL AND FINAL STATE MESONS

In pQCD calculation, the light-cone wave functions are nonperturbative and not calculable, but they are universal and channel independent for all the hadronic decays.

As a heavy meson, the B meson wave function is not well defined. In general, the B meson light-cone matrix element can be decomposed as [7]

$$\begin{aligned} &\int_0^1 \frac{d^4z}{(2\pi)^4} e^{i\mathbf{k}_1 \cdot \mathbf{z}} \langle 0 | \bar{b}_\alpha(0) d_\beta(z) | B(p_B) \rangle \\ &= -\frac{i}{\sqrt{2N_c}} \left\{ (\not{p}_B + m_B) \gamma_5 \left[\phi_B(\mathbf{k}_1) - \frac{\not{n} - \not{v}}{\sqrt{2}} \bar{\phi}_B(\mathbf{k}_1) \right] \right\}_{\beta\alpha}, \end{aligned} \tag{2}$$

where $n = (1, 0, \mathbf{0}_T)$, and $v = (0, 1, \mathbf{0}_T)$ are the unit vectors pointing to the plus and minus directions, respectively. Because the contribution of the second Lorentz structure

$\bar{\phi}_B(x, b)$ is numerically small and can be neglected, we only consider the contribution of the Lorentz structure:

$$\Phi_B(x, b) = \frac{1}{\sqrt{2N_c}} (\not{P}_B + m_B) \gamma_5 \phi_B(x, b). \quad (3)$$

In the heavy quark limit, we take the wave functions for the pseudoscalar meson D_s and the vector meson D_s^* as

$$\Phi_{D_s}(x, b) = \frac{1}{\sqrt{2N_c}} \gamma_5 (\not{P}_{D_s} + m_{D_s}) \phi_{D_s}(x, b), \quad (4)$$

$$\Phi_{D_s^*}(x, b) = \frac{1}{\sqrt{2N_c}} \not{\epsilon} (\not{P}_{D_s^*} + m_{D_s^*}) \phi_{D_s^*}(x, b), \quad (5)$$

where the polar vector $\not{\epsilon} = \frac{M_B}{\sqrt{2}M_{D_s^*}} (1, -r_{D_s^*}^2, \mathbf{0}_T)$. In the considered decays, the D_s^* meson is longitudinally polarized, so we only need to consider its wave function in longitudinal polarization.

The wave function for the light pseudoscalar meson π is given as

$$\Phi_\pi(P, x, \zeta) \equiv \frac{1}{\sqrt{2N_c}} \gamma_5 [\not{P} \phi_\pi^A(x) + m_0^\pi \phi_\pi^P(x) + \zeta m_0^\pi (\not{p} \not{n} - v \cdot n) \phi_\pi^T(x)], \quad (6)$$

where P and x are the momentum and the momentum fraction of π meson, respectively. The parameter ζ is either $+1$ or -1 depending on the assignment of the momentum fraction x . The chiral scale parameter m_0^π is defined as $m_0^\pi = m_\pi^2/(m_u + m_d)$.

III. THE PERTURBATIVE QCD CALCULATION

Using factorization theorem, we can separate the decay amplitude into soft, hard, and harder dynamics characterized by different scales, conceptually expressed as the convolution,

$$\mathcal{A}(B \rightarrow D_s^{(*)} \pi) \sim \int d^4 k_1 d^4 k_2 d^4 k_3 \text{Tr} \left[C(t) \Phi_B(k_1) \Phi_{D_s^{(*)}}(k_2) \Phi_\pi(k_3) H(k_1, k_2, k_3, t) \right], \quad (7)$$

where k_i 's are momenta of light anti-quarks included in each meson, and Tr denotes the trace over Dirac and color indices. $C(t)$ is the Wilson coefficient which results from the radiative corrections at a short distance. In the above convolution, $C(t)$ includes the harder dynamics at a larger scale than that at the M_B scale and describes the evolution of local 4-Fermi operators from m_W (the W boson mass) down to $t \sim \mathcal{O}(\sqrt{\bar{\Lambda} M_B})$ scale, where $\bar{\Lambda} \equiv M_B - m_b$. The function $H(k_1, k_2, k_3, t)$ describes the four quark operator and the spectator quark connected by a hard gluon whose q^2 is in the order of $\bar{\Lambda} M_B$, and includes the $\mathcal{O}(\sqrt{\bar{\Lambda} M_B})$ hard dynamics. Therefore, this hard part H can be perturbatively calculated. The function $\Phi_{(D_s^{(*)}, \pi)}$ are the wave functions of $D_s^{(*)}$ and π .

Since the b quark is rather heavy, we consider the B meson at rest for simplicity. It is convenient to use the light-cone coordinate (p^+, p^-, \mathbf{p}_T) is used to describe the meson's momenta:

$$p^\pm = \frac{1}{\sqrt{2}} (p^0 \pm p^3), \quad \text{and} \quad \mathbf{p}_T = (p^1, p^2). \quad (8)$$

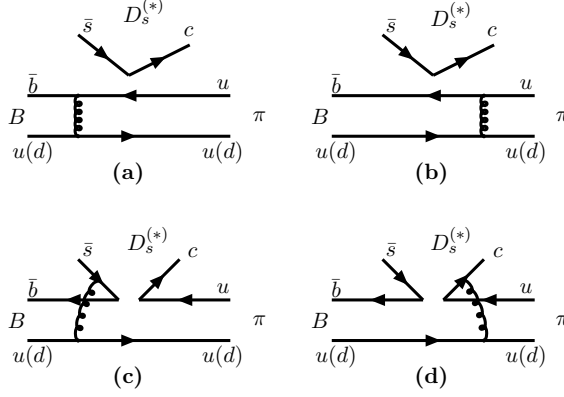


FIG. 1: Diagrams contributing to the decays $B \rightarrow D_s^{(*)}\pi$.

At the rest frame of B meson, the light meson moves very fast and so P_3^+ or P_3^- can be treated as zero. Using these coordinates, the B meson and the two final state meson momenta can be written as

$$P_B = \frac{M_B}{\sqrt{2}}(1, 1, \mathbf{0}_T), \quad P_2 = \frac{M_B}{\sqrt{2}}(1, r^2, \mathbf{0}_T), \quad P_3 = \frac{M_B}{\sqrt{2}}(0, 1 - r^2, \mathbf{0}_T), \quad (9)$$

respectively, where $r = M_{D_s^{(*)}}/M_B$. Putting the light anti-quark momenta in B , $D_s^{(*)}$ and π mesons as k_1 , k_2 , and k_3 , respectively, we can choose

$$k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T}), \quad k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2T}), \quad k_3 = (0, x_3 P_3^-, \mathbf{k}_{3T}). \quad (10)$$

For these considered decay channels, the integration over k_1^- , k_2^- , and k_3^+ in equation (7) will lead to

$$\mathcal{A}(B \rightarrow D_s^{(*)}\pi) \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \cdot \text{Tr} \left[C(t) \Phi_B(x_1, b_1) \Phi_{D_s^{(*)}}(x_2, b_2) \Phi_\pi(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)} \right], \quad (11)$$

where b_i is the conjugate space coordinate of k_{iT} , and t is the largest energy scale in the function $H(x_i, b_i, t)$. The last term $e^{-S(t)}$ in equation (11) is the Sudakov form factor which suppresses the soft dynamics effectively [8].

For the considered decays, the related weak effective Hamiltonian H_{eff} can be written as [9]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} [(C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu))], \quad (12)$$

where the four-quark operators are

$$O_1 = (\bar{b}_\alpha u_\beta)_{V-A} (\bar{c}_\alpha s_\beta)_{V-A}, \quad O_2 = (\bar{b}_\alpha u_\alpha)_{V-A} (\bar{c}_\alpha s_\alpha)_{V-A}, \quad (13)$$

with α, β being the color indexes, and $(\bar{q}_1 q_2)_{V-A} = \bar{q}_1 \gamma^\mu (1 - \gamma^5) q_2$. The Fermi constant $G_F = 1.16639 \times 10^{-5} GeV^{-2}$ and $C_{1,2}(\mu)$ are Wilson coefficients running with the renormalization scale μ . The leading order diagrams contributing to the decays $B \rightarrow D_s^{(*)}\pi$ are drawn in figure 1 according to this effective Hamiltonian.

In the following, we will get the analytic formulas by calculating the hard part $H(t)$ at leading order. Involving the meson wave functions, the amplitude for the factorizable tree emission diagrams Fig.1(a) and (b) can be written as:

$$\begin{aligned}
F_e = & 8\pi C_F f_{D_s^{(*)}} \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \Phi_B(x_1, b_1) \\
& \times \left\{ [(x_3 + 1)\phi_\pi^A(x_3) - r_\pi(2x_3 - 1)(\phi_\pi^P(x_3) + \phi_\pi^T(x_3))] \right. \\
& \times E_e(t) h_e(x_1, x_3(1 - r_{D_s^{(*)}}^2), b_1, b_3) S_t(x_3) \\
& \left. + 2r_\pi \phi_\pi^P(x_3) E_e(t') h_e(x_3, x_1(1 - r_{D_s^{(*)}}^2), b_3, b_1) S_t(x_1) \right\}, \quad (14)
\end{aligned}$$

where $C_F = 4/3$ is the group factor of $SU(3)_c$ gauge group, and the mass ratios $r_\pi = m_\pi/m_B$, $r_{D_s^{(*)}} = m_{D_s^{(*)}}/m_B$. Here $f_{D_s^{(*)}}$ is the decay constant of $D_s^{(*)}$ meson, and $S_t(x)$ is the jet function [10]. The factor evolving with the scale t is given by:

$$E_e(t) = \alpha_s(t) \exp[-S_B(t) - S_\pi(t)], \quad (15)$$

where $S_B(t), S_\pi(t)$ are expressions for Sudakov form factors [10]. The hard function is written as

$$\begin{aligned}
h_e(x_1, x_2, b_1, b_2) = & K_0(\sqrt{x_1 x_2} m_B b_1) [\theta(b_1 - b_2) K_0(\sqrt{x_2} m_B b_1) I_0(\sqrt{x_2} m_B b_2) \\
& + \theta(b_2 - b_1) K_0(\sqrt{x_2} m_B b_2) I_0(\sqrt{x_2} m_B b_1)]. \quad (16)
\end{aligned}$$

The hard scales $t^{(\prime)}$ in Eq.(14) are determined by

$$\begin{aligned}
t = & \max(\sqrt{x_3(1 - r_{D_s^{(*)}}^2)} m_B, 1/b_1, 1/b_3), \\
t' = & \max(\sqrt{x_1(1 - r_{D_s^{(*)}}^2)} m_B, 1/b_1, 1/b_3). \quad (17)
\end{aligned}$$

For the nonfactorizable tree emission diagrams Fig.1(c) and (d), all three meson wave functions are involved. The integraton of b_3 can be performed using δ function $\delta(b_3 - b_2)$ and the result is

$$\begin{aligned}
M_e = & -16\pi \sqrt{2N_c} C_F \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_B(x_1, b_1) \Phi_{D_s^{(*)}}(x_2) \\
& \times \left\{ [(x_2 - 1)\phi_\pi^A(x_3) + r_\pi x_3(\phi_\pi^P(x_3) - \phi_\pi^T(x_3))] E_n(t) h_n^1(x_1, x_2, x_3, b_1, b_2) \right. \\
& \left. + [(x_3 + x_2)\phi_\pi^A(x_3) - r_\pi x_3(\phi_\pi^P(x_3) + \phi_\pi^T(x_3))] E_n(t') h_n^2(x_1, x_2, x_3, b_1, b_2) \right\}, \quad (18)
\end{aligned}$$

where the expressions for the evolution factor is $E_n = \alpha_s(t) \exp[-S(t)|_{b_3=b_1}]$ with the Sudakov exponent $S = S_B + S_{D_s^{(*)}} + S_\pi$.

The hard functions $h_n^i, i = 1, 2$ in the amplitude are given as

$$\begin{aligned}
h_n^i = & [\theta(b_1 - b_2) K_0(Ab_1) I_0(Ab_2) + \theta(b_2 - b_1) K_0(Ab_2) I_0(Ab_1)] \\
& \times \begin{pmatrix} \frac{\pi i}{2} H_0(\sqrt{|G_i^2|} b_3), & \text{for } G_i^2 < 0 \\ K_0(G_i b_2), & \text{for } G_i^2 > 0 \end{pmatrix}, \quad (19)
\end{aligned}$$

TABLE I: Input parameters used in the numerical calculation[6, 11].

Masses	$m_\pi = 0.14 \text{ GeV},$ $m_{D_s} = 1.9685 \text{ GeV},$ $m_B = 5.28 \text{ GeV},$	$m_0^\pi = 1.3 \text{ GeV},$ $m_{D_s^*} = 2.1123 \text{ GeV},$ $m_W = 80.4 \text{ GeV},$
Decay constants	$f_B = 0.19 \text{ GeV},$ $f_{D_s} = 0.273 \text{ GeV},$	$f_\pi = 0.13 \text{ GeV},$ $f_{D_s^*} = 0.312 \text{ GeV},$
Lifetimes	$\tau_{B^\pm} = 1.638 \times 10^{-12} \text{ s}, \tau_{B^0} = 1.530 \times 10^{-12} \text{ s},$	
<i>CKM</i>	$V_{cb} = 0.0412 \pm 0.0011, V_{us} = 0.2255 \pm 0.0019.$	

with the variables

$$\begin{aligned}
A^2 &= x_1 x_3 (1 - r_{D_s^{(*)}}^2) m_B^2, \\
G_1^2 &= (x_1 + x_2) r_{D_s^{(*)}}^2 - (1 - x_1 - x_2) x_3 (1 - r_{D_s^{(*)}}^2) m_B^2, \\
G_2^2 &= (x_1 - x_2) x_3 (1 - r_{D_s^{(*)}}^2) m_B^2.
\end{aligned} \tag{20}$$

The hard scales in Eq.(19) are given by

$$\begin{aligned}
t &= \max(Am_B, \sqrt{G_1^2} m_B, 1/b_1, 1/b_2), \\
t' &= \max(Am_B, \sqrt{G_2^2} m_B, 1/b_1, 1/b_2).
\end{aligned} \tag{21}$$

Then the total decay amplitude of $B \rightarrow D_s^{(*)} \pi$ decays can be written as

$$\mathcal{A}(B \rightarrow D_s^{(*)} \pi) = V_{ub}^* V_{cs} [F_e (C_2 + \frac{C_1}{3}) + M_e C_1]. \tag{22}$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

For the numerical calculation, we list the input parameters in Table I.

For the B meson wave function, we adopt the model

$$\phi_B(x, b) = N_B x^2 (1 - x)^2 \exp \left[-\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2} (\omega_b b)^2 \right], \tag{23}$$

where ω_b is a free parameter and we take $\omega_b = 0.4 \pm 0.04 \text{ GeV}$ in numerical calculations, and $N_B = 91.745$ is the normalization factor for $\omega_b = 0.4$.

For $D_s^{(*)}$ meson, the distribution amplitude is taken as:

$$\phi_{D_s^{(*)}}(x) = f_{D_s^{(*)}} \frac{1}{\sqrt{6}} x(1 - x) \left[1 - a_{D_s^{(*)}} (1 - 2x) \right], \tag{24}$$

with the Gegenbauer coefficients $a_{D_s} = 0.3$ and $a_{D_s^*} = 0.78$. The CLEO and BarBar collaborations reported their work on the measurements of the decay constant of D_s

meson and obtained $f_{D_s} = 274 \pm 13 \pm 7$ MeV [11] and $283 \pm 17 \pm 7 \pm 14$ MeV [12], respectively. However, the decay constant of the vector meson D_s^* has not been directly measured in experiments so far. From the conclusions draw by the CLEO collaboration [11], one can find that there exists a relation:

$$\frac{f_{D_s^*}}{f_{D^*}} \approx \frac{f_{D_s}}{f_D} \approx \frac{f_{B_s}}{f_B} = [1.1, 1.2], \quad (25)$$

which is consistent with that from lattice simulation [13] and the QCD sum rules calculations [14]. From table I, it is easy to see the value of the ratio $f_{D_s^*}/f_{D_s}$ is 1.14 in our work. It is different from [15], where the relation between $f_{D_s^*}$ and f_{D_s} derived from HQET

$$\frac{f_{D_s^*}}{f_{D_s}} = \sqrt{\frac{m_{D_s}}{m_{D_s^*}}}, \quad (26)$$

was used. From this equation, one can get the value of $f_{D_s^*}$, which is smaller than that of f_{D_s} .

The twist-2 pion distribution amplitude ϕ_π^A , and the twist-3 ones ϕ_π^P and ϕ_π^T have been parametrized as

$$\begin{aligned} \phi_\pi^A(x) = & \frac{f_\pi}{2\sqrt{2N_c}} 6x(1-x) \left[1 + a_1^\pi C_1^{3/2}(2x-1) + a_2^\pi C_2^{3/2}(2x-1) \right. \\ & \left. + a_4^\pi C_4^{3/2}(2x-1) \right], \end{aligned} \quad (27)$$

$$\begin{aligned} \phi_\pi^P(x) = & \frac{f_\pi}{2\sqrt{2N_c}} \left[1 + (30\eta_3 - \frac{5}{2}\rho_\pi^2) C_2^{1/2}(2x-1) - 3 \left\{ \eta_3 \omega_3 + \frac{9}{20} \rho_\pi^2 (1 + 6a_2^\pi) \right\} \right. \\ & \left. \times C_4^{1/2}(2x-1) \right], \end{aligned} \quad (28)$$

$$\phi_\pi^T(x) = \frac{f_\pi}{2\sqrt{2N_c}} (1-2x) \left[1 + 6(5\eta_3 - \frac{1}{2}\eta_3 \omega_3 - \frac{7}{20}\rho_\pi^2 - \frac{3}{5}\rho_\pi^2 a_2^\pi)(1-10x+10x^2) \right], \quad (29)$$

with the mass ratio $\rho_\pi = (m_u + m_d)/m_\pi = m_\pi/m_0^\pi$ and the Gegenbauer polynomials $C_n^\nu(t)$,

$$C_2^{1/2}(t) = \frac{1}{2}(3t^2 - 1), \quad C_4^{1/2}(t) = \frac{1}{8}(3 - 30t^2 + 35t^4), \quad (30)$$

$$C_1^{3/2}(t) = 3t, \quad C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1), \quad (31)$$

$$C_4^{3/2}(t) = \frac{15}{8}(1 - 14t^2 + 21t^4). \quad (32)$$

The Gegenbauer coefficients are given as

$$a_1^\pi = 0, \quad a_2^\pi = 0.115, \quad a_4^\pi = -0.015. \quad (33)$$

The values of other parameters are taken as [16] $\eta_3 = 0.015$ and $\omega = -3.0$.

In the B-rest frame, the decay width of $B \rightarrow D_s^{(*)} \pi$ can be obtained by

$$\Gamma = \frac{1}{32\pi} G_F^2 m_B^7 |\mathcal{A}|^2 (1 - r_{D_s^{(*)}}^2), \quad (34)$$

TABLE II: Branching ratios ($\times 10^{-5}$) for the decays $B^0 \rightarrow D_s^+ \pi^-$, $D_s^{*+} \pi^-$ and $B^+ \rightarrow D_s^+ \pi^0$, $D_s^{*+} \pi^0$. The first theoretical error is from the the B meson shape parameter ω_b . The second error is from the higher order pQCD correction. The third one is from the uncertainties of CKM matrix elements.

Channel	This work	Data
$B^0 \rightarrow D_s^+ \pi^-$	$1.85^{+0.36+0.41+0.10}_{-0.52-0.56-0.10}$	1.53 ± 0.35
$B^+ \rightarrow D_s^+ \pi^0$	$1.98^{+0.39+0.81+0.11}_{-0.56-0.31-0.11}$	1.6 ± 0.6
$B^0 \rightarrow D_s^{*+} \pi^-$	$2.59^{+0.45+0.70+0.15}_{-0.76-0.60-0.15}$	3.0 ± 0.7
$B^+ \rightarrow D_s^{*+} \pi^0$	$2.78^{+0.48+0.74+0.16}_{-0.82-0.65-0.16}$	< 27

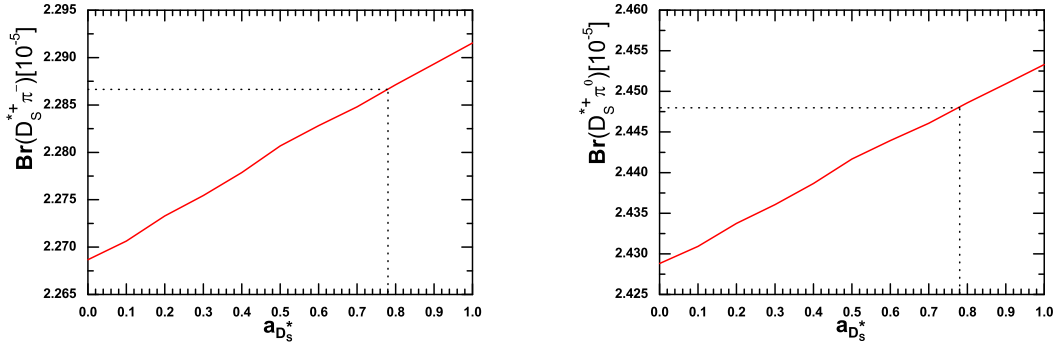


FIG. 2: Branching ratios (in units of 10^{-5}) of $B^0 \rightarrow D_s^{*+} \pi^-$ and $B^+ \rightarrow D_s^{*+} \pi^0$ decays as functions of Gegenbauer moment $a_{D_s^*}$.

where \mathcal{A} is the total decay amplitude shown in Eq.(22).

Using the wave functions as specified in the previous section and the input parameters listed in this section, it is straightforward to calculate the CP-averaged branching ratios for the considered decays, which are listed in Table II. The first error in these entries is caused by the B meson shape parameter $\omega_b = 0.40 \pm 0.04$. The second error is from the higher order pQCD correction: the choice of hard scales, defined in Eq.(17) and Eq.(21), which vary from $0.9t$ to $1.1t$. The third error is from the uncertainties of the CKM matrix elements which are listed in table I.

In previous calculations [1, 2], the authors have considered that the value of the Gegenbauer moment $a_{D_s^*}$ was the same as that of a_{D_s} and taken them as 0.3. Here we take $a_{D_s^*} = 0.78$, which is determined to fit the requirement that $\phi_{D_s^*}(x)$, shown in Eq.(24), has a maximum at $\bar{x} = \frac{m_{D_s} - m_c}{m_{D_s}}$. In Fig. 2, we plot that $a_{D_s^*}$ dependence of the branching ratios of $B^0 \rightarrow D_s^{*+} \pi^-$ and $B^+ \rightarrow D_s^{*+} \pi^0$. One can find that the branching ratios are not sensitive to the variations of $a_{D_s^*}$.

From the numerical results, we find that the non-factorizable contributions are very

small and almost neglectable. They are about 10% of the factorizable ones in each decays. The main contributions come from the factorizable amplitudes.

V. CONCLUSION

In this paper, we calculate the branching ratios of decays $B^0 \rightarrow D_s^+ \pi^-$, $B^+ \rightarrow D_s^+ \pi^0$, $B^0 \rightarrow D_s^{*+} \pi^-$ and $B^+ \rightarrow D_s^{*+} \pi^0$ in the pQCD factorization approach. We find that:

- The decays considered here have branching ratios about 10^2 smaller than those of the $B \rightarrow D^{(*)} \pi$ decays, and they comes mainly from the relevant CKM matrix elements.
- From the numerical results shown in table II, one can find that the pQCD predictions for these considered decay channels are consistent with the measured values and currently available experimental upper limit.
- To determine decay constant of the vector meson D_s^{*+} , the relation

$$\frac{f_{D_s^*}}{f_{D^*}} \approx \frac{f_{D_s}}{f_D} \approx \frac{f_{B_s}}{f_B} \quad (35)$$

is used. It indicates that the value of $f_{D_s^*}$ is larger than that of f_{D^*} , which is contrary to the conclusion derived from the relation

$$\frac{f_{D_s^*}}{f_{D_s}} = \sqrt{\frac{m_{D_s}}{m_{D_s^*}}} . \quad (36)$$

- In the numerical calculation, we take $a_{D_s^*} = 0.78$, which is larger than the value given in the previous calculations. It is determined to fit the requirement that the wave function $\phi_{D_s^*}(x)$ has a maximum at $\bar{x} = \frac{m_{D_s} - m_c}{m_{D_s}}$.

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